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Neutron bound S-states in Woods-Saxon potential

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The S-wave bound states of neutrons in Woods-Saxon potential have been investigated by calculating the zeros of S_0-matrix on the negative imaginary axis of the complex k-plane. Numerical results are presented for the atomic mass $A = 200$.

Introduction

In the choice of a suitable potential between nucleon and the nucleus, which may take proper account of the general properties of nuclear energy levels, emphasis has so far been given to the analytical convenience. The square-well and harmonic oscillator potentials are quite appealing in this respect since they are amenable to analytical treatment. However, these potentials have severe limitations. The harmonic oscillator potential becomes asymptotically infinite and yields an infinite set of discrete eigenvalues, on the other hand, the square-well potential shows an abrupt change of nuclear potential which is not very realistic. Besides, neither of these potentials gives the proper level sequence. Investigations by Feenberg (1950), Malenka (1952) and others suggest that better level ordering can be obtained if the boundary of the nuclear potential is taken to be diffuse. Malenka has studied the problem of bound states of a heavy nucleus with a potential which has precisely the analytic form of the square-well in the interior region and exponentially diffused form in external region. Green & Lee (1955) have investigated systematically the energy eigenvalue problem with a similar type of spherical-well potential having an exponentially diffused boundary and solved the Schrödinger equation analytically. But these authors have also taken account of the higher angular momentum states by suitably approximating the centrifugal term The success of the most realistic diffuse potential of Woods-Saxon form (1954) in the scattering problem (Feshbach 1958, Beyster *et al* 1956) have stimulated a number of authors (Ross *et al* 1956, Nemirovskii 1958 and Ghosh & Sil 1960) to apply the same also to the investigations of the bound states of nucleons. Though this form avoids the unrealistic sharp corner of the exponentially diffuse edge potential, it brings some complications in the exact analytical treatment, so that most of its applications have so far been done by numerical integration of the wave equation. Ross *et al* (1956) have used this potential to study numerically some of the upper states of nucleons in nuclei with a spin-orbit interaction, Nemirovskii (1958) has also dealt numerically the problem of bound states of a neutron in Woods-Saxon potential including the spin-orbit interaction. The analytic solution for the S-states as given by Lawson (1956) is not of much practical value because of very slow convergence of the series solution. Ghosh & Sil (1960) have solved the wave equation with the above potential by applying the method of Lanczos (1938). They have considered higher angular momentum states with a suitable approximation for the centrifugal term.

In the present paper we have calculated S-state energy levels for the atomic mass $A = 200$ with Woods-Saxon potential using the expression for the S_0 matrix as given by Bencze (1966). The results of Ghosh & Sil agree fairly well with those of ours.

Theory and Formulation

The interaction potential between the neutron and the nucleus, which is taken to be of Woods-Saxon form, is represented as

$$V(r) = -V_0/[1+e^{(r-R)/a}]$$

where R is the nuclear radius and a the diffusivity parameter. The analytic expression for the S_0-matrix element with this potential, following Bencze (1966), is

$$S_0(k) = e^{-2kiR}\cdot\frac{\Gamma(2\iota ka)}{\Gamma(-2\iota ka)}\times$$
$$\frac{A\dfrac{\Gamma(1-2\lambda)}{\Gamma(1-\lambda+\iota ka)\Gamma(-\lambda+\iota ka)}-\dfrac{\Gamma(1+2\lambda)}{\Gamma(\lambda+\iota ka)\Gamma(1+\lambda+\iota ka)}}{\dfrac{\Gamma(1+2\lambda)}{\Gamma(1+\lambda-\iota ka)\Gamma(\lambda-\iota ka)}-A\dfrac{\Gamma(1-2\lambda)}{\Gamma(-\lambda-\iota ka)\Gamma(1-\lambda-\iota ka)}} \tag{1}$$

$$\text{where}\quad A = \left(\frac{b}{1+b}\right)^{2\lambda}(1+b)^{-2\iota ka}\,\frac{{}_2F_1\left(\lambda+\iota ka,\,1+\lambda+\iota ka,\,1+2\lambda;\,\dfrac{b}{1+b}\right)}{{}_2F_1\left((-\lambda-\iota ka,\,1-\lambda-\iota ka,\,1-2\lambda;\,\dfrac{b}{1+b}\right)}$$

$$\lambda = \pm\,\iota\sqrt{k^2+k_0^2}$$

$$k^2 = \frac{2M}{\hbar^2}E$$

$$k_0^2 = \frac{2M}{\hbar^2}V_0 \qquad \text{and} \qquad b = \exp(-R/a)$$

M and E being respectively, the mass and energy of the neutron. $S_0(k)$ may be written as (Mott & Massey, 1965)

$$S_0(k) = f_0(k)/f_0(-k) \qquad \ldots (2)$$

where $f_0(k) = \lim_{\gamma \to 0} f_0(k, r)$ and $f_0(\pm k, r)$ are linearly independent solutions of the radial wave equation for the scattering of S-wave neutrons

$$\frac{d^2 u_0}{\hbar dr^2} + \left[k^2 - \frac{2M}{\hbar^2} V(r) \right] u_0(r) = 0 \qquad \dots (3)$$

and have the asymptotic form

$$f_0(\pm k, r) \sim \exp(\pm \iota kr)$$

The S-wave bound states are determined by the zeros of $S_0(k)$ corresponding to the zeros of $f_0(k)$ on the negative imaginary axis of the complex k-plane. The redundant zeroes of $S_0(k)$ on account of the poles of $f_0(-k)$ on the negative imaginary axis need not be taken into consideration, since these poles do not correspond to the true bound states (Mott & Massey 1965). For numerical computation we note that $b \sim 10^{-6} b \ll 1$ for the mass number considered, so that the hypergeometric functions reduced to 1 and $(1+b) \to 1$. Writing $k = -\iota\kappa$ where κ is positive, we have from (1) and (2) for a bound state at $-\frac{\hbar^2}{2M}\kappa^2$,

$$C - C^* = 0, \text{ i.e., } Im C = 0 \qquad \dots (4)$$

where $$C = \frac{b^\lambda \Gamma(1-2\lambda)}{\Gamma(1-\lambda+\kappa a)\,\Gamma(-\lambda+\kappa a)}$$

With a simple algebraic manipulation we obtain from equation (4)

$$\cot pR = \eta, \text{ where } \eta = \frac{\mu \cos y + \delta \sin y}{\mu \sin y - \delta \cos y}, \qquad \dots (5)$$

in which $$p = \sqrt{\frac{2M}{\hbar^2}(V_0 - |E|)},\ \delta = pa,\ \mu = \kappa a,$$

and $$y = Im[\log \Gamma(1-2\iota\delta) - 2 \log \Gamma(1+\mu-\iota\delta)].$$

For the square-well case $a \to 0$. Then we have $y \propto \delta$, and hence

$$\cot pR = -\frac{\mu}{\delta} = -\frac{\kappa}{p}$$

or $$p \cot pR = -\kappa. \qquad \dots (6)$$

The expression (6) is identical with the analytic form given by Schiff (1955). The required eigenvalues are found by solving the transcendental equation (5).

Results and Discussion

To calculate the S-state energy levels for the atomic mass $A = 200$, we choose the values of the parameters in the potential function as $V_0 = 52$ Mev, $R = 1{\cdot}25\ A^{1/3} fm$ and $a = 0{\cdot}52\, fm$, which are the same as taken by us in the calculation of the neutron strength function (1967).

Table 1

Levels	$\lvert E \rvert$ in Mev	
	Our results	Results due to Ghosh & Sil (1960)
1s	47.98	48.1
2s	36.99	35.5
3s	21 09	20.2
4s	3.69	4.8

In figure 1 we have plotted cot pR and η occuring in equation (5) as functions of δ. The points of intersection of these two curves give the required energy eigenvalues which are shown in table 1. For comparison we have also given the

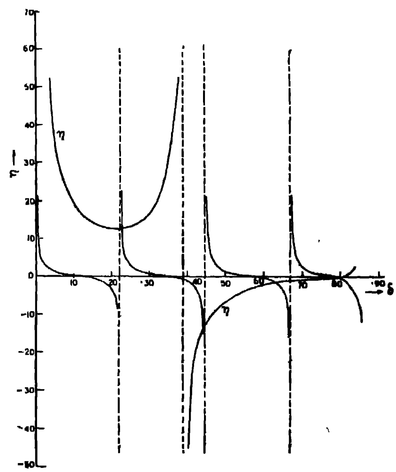

Figure 1.

corresponding eigenvalues as calculated from the energy level diagram given by Ghosh & Sil (1960). The values of the parameters used by these authors are

$$V_0 = 52 \text{ Mev}, \qquad R = (1\cdot 15\, A^{1/3}+0\cdot 4)\times 10^{-13} \text{ cm.}$$

and

$$a = 0\cdot 57\times 10^{-13} \text{ cm.}$$

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